Population Size Effects in Structural Development

Oksana Leukhina and Stephen Turnovsky

Paris, 2015

Pre-crisis Observations

1. Motivating Evidence

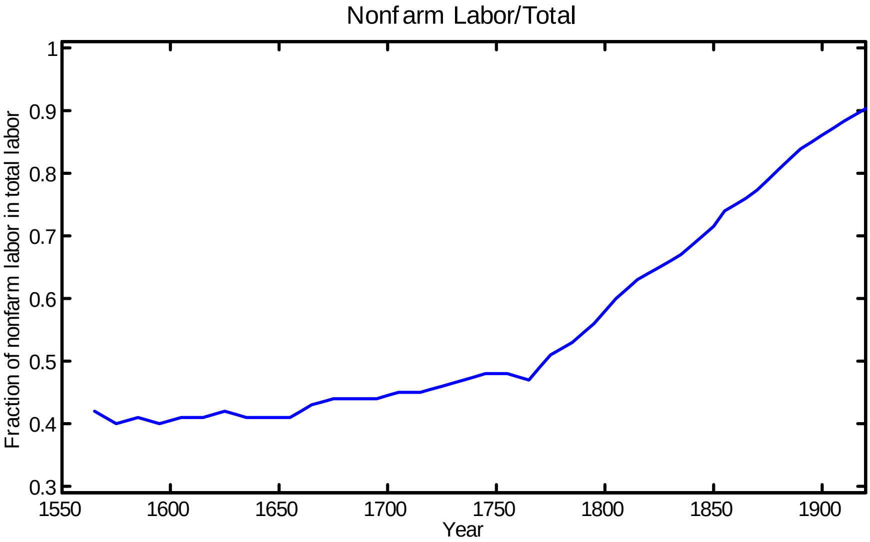
Pre-crisis Observations

1. Motivating Evidence
2. Analytical Framework and Results

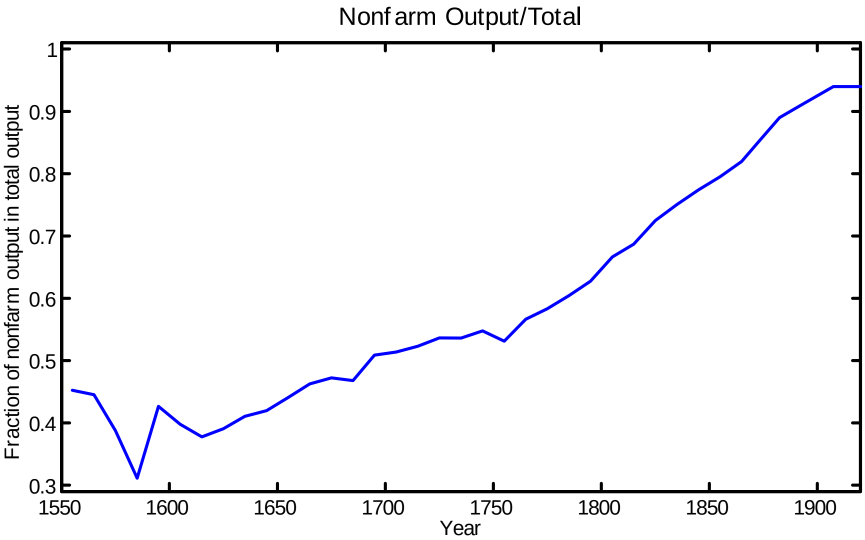
Pre-crisis Observations

1. Motivating Evidence
2. Analytical Framework and Results
3. Quantitative Investigation of Structural Development of England: 1650-1920

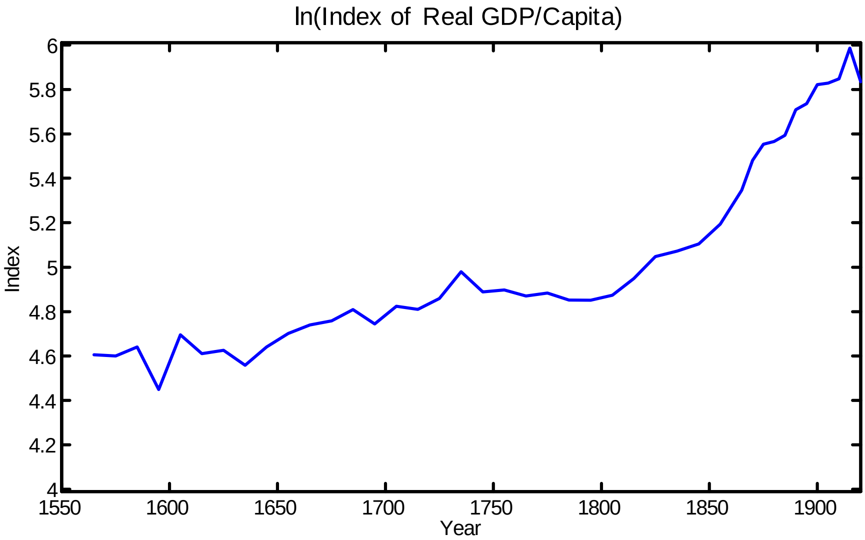
Structural Development of England



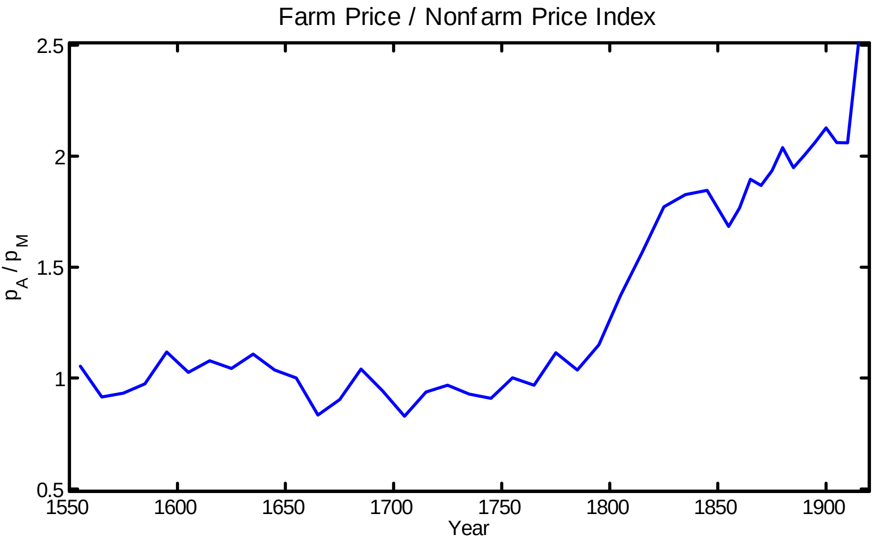
Structural Development of England



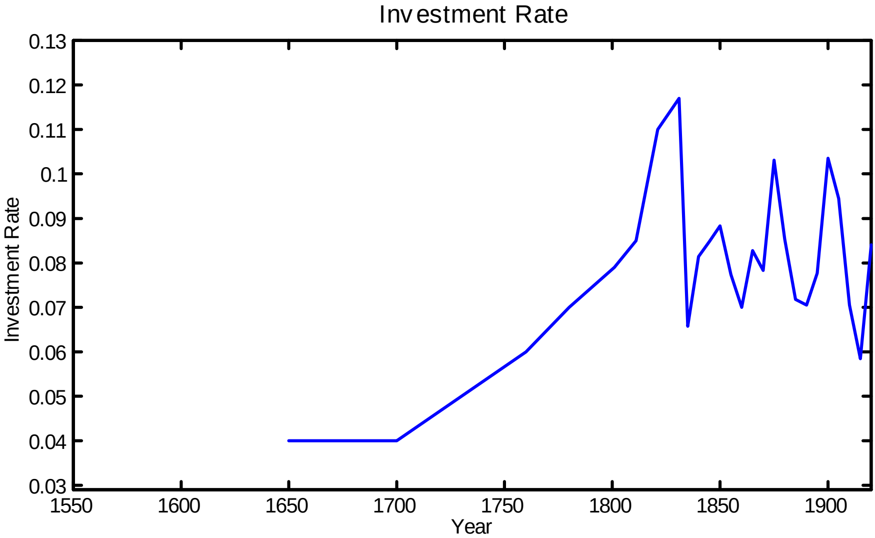
Structural Development of England



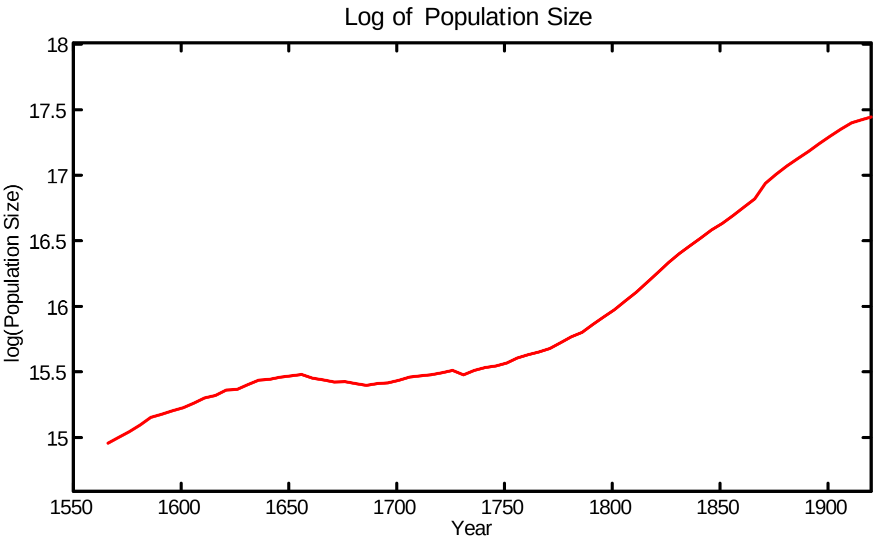
Structural Development of England



Structural Development of England



Structural Development of England



Existing Explanations of Structural Development

- Typically focus on a very specific channel
  - gross substitutes: faster productivity growth sector pulls labor in [e.g. Lewis (1954), Hansen and Prescott (2000), Doepke (2004), Bar and Leukhina (2008)]
  - non-homotheticity: farm productivity growth pushes labor out [Murphy et al. ,1989, Matsuyama, 1992, Laitner, 2000, Caselli and Coleman, 2001, Gollin et al., 2002, Voigtländer and Voth, 2006]

Shortcomings

- No consensus on the most appropriate channel or combination
- Population size effects have not been properly investigated
  - Two references in the literature to population size effects
    1. Gollin and Rogerson (JED, 2014) highlight the adverse effect of population size on L_m/L due to the presence of subsistence consumption.
    2. Bar and Leukhina (RED, 2010) highlight the positive effect of population size on L_m/L due to a greater intensity of labor in manufacturing.
  - However, in many models of structural development, population size enters the model in a neutral way. We will argue such frameworks are empirically implausible.

What We Do

- Examine analytically the effects of population size on structural development in a parsimonious general equilibrium two sector growth model.
  - Important: general production functions, general utility function.
- Study the effects of population change on structural development in the case of England.

Model

- Standard Dynamic GE model
- There is a large number of identical infinitely-lived families of mass 1, each composed of L_t identical individuals at time t.
- Two types of goods:

1. Farm good is produced with capital, labor and land
2. Nonfarm good is produced with capital and labor only.

Production

Nonfarm good is numeraire
F, G are homogeneous of degree 1

- Nonfarm sector solves

$$\max_{K_{M,t},L_{M,t}} F_t\left(K_{M,t}, L_{M,t}\right) - w_t L_{M,t} - r_t K_{M,t}$$

Production

Nonfarm good is numeraire

F, G are homogeneous of degree 1

- Nonfarm sector solves

$$\max_{K_{M,t},L_{M,t}} F_t\left(K_{M,t}, L_{M,t}\right) - w_t L_{M,t} - r_t K_{M,t}$$

- Farm sector solves

$$\max_{K_{A,t},L_{A,t},\Lambda_t} p_t G_t\left(K_{A,t}, L_{A,t}, \Lambda_t\right) - w_t L_{A,t} - r_t K_{A,t} - \rho_t \Lambda_t$$

Families

Given $\{w_t, r_t, \rho_t, p_t\}_{t=0}^{\infty}$, families make consumption and capital accumulation choices to solve

$$\max_{\{a_t, c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t u(a_t - \bar{a}, c_t) \text{ subject to}$$

$$p_t a_t + c_t + k_{t+1} \frac{L_{t+1}}{L_t} - (1-\delta) k_t = r_t k_t + w_t + \rho_t \lambda_t, \text{ for all } t$$

$$a_t, c_t, k_{t+1} \geq 0, \text{ for all } t$$

$$k_0, \lambda_0 \text{ given}$$

Trade

To capture the expansion of trade, we assume that imports of food grow in accordance with the data, $\{M_t\}$

The amount of exports $\{X_t\}$ adjusts endogenously to ensure a trade balance,

$$X_t = p_t M_t.$$

Market Clearing

$$
\begin{aligned}
L_{M,t} + L_{A,t} &= L_t, \\
K_{M,t} + K_{A,t} &= K_t. \\
\Lambda_t &= \Lambda. \\
C_t + K_{t+1} - (1-\delta)K_t + X_t &= F(K_{M,t}, L_{M,t}, t). \\
A_t &= G_t(K_{A,t}, L_{A,t}, \Lambda_t, t) + M_t.
\end{aligned}
$$

Competitive Equilibrium

A competitive equilibrium consists of allocations $\{A_t, C_t, K_{t+1}, K_{A,t}, K_{M,t}, L_{A,t}, L_{M,t}, \Lambda_t, X_t\}$ and prices $\{p_t, w_t, r_t, \rho_t\}$ such that

- firms' maximization problems are solved
- families' maximization problem is solved
- market clearing and trade balance conditions are satisfied.

Equilibrium Characterization

Families' maximization is characterized by

$$p_t = \frac{u_2(c_t, a_t)}{u_1(c_t, a_t)},$$

$$\beta(r_{t+1} + 1 - \delta) = \frac{u_1(c_t, a_t)}{u_1(c_{t+1}, a_{t+1})}.$$

Profit maximization is characterized by

$$r_t = p_t G_1(K_{A,t}, L_{A,t}, \Lambda_t, t) = F_1(K_{M,t}, L_{M,t}, t),$$

$$w_t = p_t G_2(K_{A,t}, L_{A,t}, \Lambda_t, t) = F_2(K_{M,t}, L_{M,t}, t),$$

$$\rho_t = p_t G_3(K_{A,t}, L_{A,t}, \Lambda_t, t).$$

Market clearing from above.

Functional Forms

- Production technology

$$
\begin{aligned}
G_t(K_{A,t}, L_{A,t}, \Lambda_t) &\equiv B_{A,t}\left[b_{A,t}K_{A,t}^{\varepsilon} + c_{A,t}L_{A,t}^{\varepsilon} + d_{A,t}\Lambda_t^{\varepsilon}\right]^{\frac{1}{\varepsilon}} \\
F_t(K_{M,t}, L_{M,t}) &\equiv B_{M,t}K_{M,t}^{\upsilon}L_{M,t}^{1-\upsilon}
\end{aligned}
$$

Functional Forms

- Production technology

$$G_t(K_{A,t}, L_{A,t}, \Lambda_t) \equiv B_{A,t}\left[b_{A,t}K_{A,t}^{\varepsilon} + c_{A,t}L_{A,t}^{\varepsilon} + d_{A,t}\Lambda_t^{\varepsilon}\right]^{\frac{1}{\varepsilon}}$$

$$F_t(K_{M,t}, L_{M,t}) \equiv B_{M,t}K_{M,t}^{\upsilon}L_{M,t}^{1-\upsilon}$$

- Utility

$$U(a_t - \bar{a}, c_t) = \frac{1}{\gamma}\left[\alpha\left(a_t - \bar{a}L_t\right)^{\rho} + (1-\alpha)\left(c_t\right)^{\rho}\right]^{\frac{\gamma}{\rho}}$$

Population Effect

Can be seen in the intratemporal tradeoff equation

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{G\left(K_{A,t},L_{A,t},\Lambda_t,t\right)-\bar{a}L_t}{F(K_{M,t},L_{M,t},t)-I_t-X_t}\right)^{\rho-1}=p_t=\frac{F_2\left(K_{M,t},L_{M,t},t\right)}{G_2\left(K_{A,t},L_{A,t},\Lambda_t,t\right)}$$

It depends crucially upon

- Elasticities of substitution in farm production $\frac{1}{1-\varepsilon}$

Population Effect

Can be seen in the intratemporal tradeoff equation

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{G\left(K_{A,t}, L_{A,t}, \Lambda_t, t\right)-\bar{a}L_t}{F(K_{M,t}, L_{M,t}, t)-I_t-X_t}\right)^{\rho-1}=p_t=\frac{F_2\left(K_{M,t}, L_{M,t}, t\right)}{G_2\left(K_{A,t}, L_{A,t}, \Lambda_t, t\right)}$$

It depends crucially upon

- Elasticities of substitution in farm production $\frac{1}{1-\varepsilon}$
- Elasticity of substitution in utility $\frac{1}{1-\rho}$

Population Effect

Can be seen in the intratemporal tradeoff equation

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{G\left(K_{A,t}, L_{A,t}, \Lambda_t, t\right)-\bar{a}L_t}{F(K_{M,t}, L_{M,t}, t)-I_t-X_t}\right)^{\rho-1}=p_t=\frac{F_2\left(K_{M,t}, L_{M,t}, t\right)}{G_2\left(K_{A,t}, L_{A,t}, \Lambda_t, t\right)}$$

It depends crucially upon

- Elasticities of substitution in farm production $\frac{1}{1-\varepsilon}$
- Elasticity of substitution in utility $\frac{1}{1-\rho}$
- Expansion of trade

Employment Share in Manufacturing: Analytical Results

- Population Size

$$\frac{d\hat{L}_M}{d\hat{L}} - 1 = \frac{\tilde{L}_A a_\Lambda}{\kappa}\left[(1-\theta)\left(\frac{1}{\frac{1}{1-\varepsilon}} - \frac{1}{\frac{1}{1-\rho}(1-\chi)}\right) - \frac{\theta}{\frac{1}{1-\rho}}\left(1 - \frac{1}{\frac{1}{1-\varepsilon}}\right)\right]$$

a_j – farm income shares, $\chi = \bar{a}\tilde{L}/\tilde{A}$, $\theta = p\tilde{M}/\tilde{Y}_M$

Employment Share in Manufacturing: Analytical Results

- Population Size

$$\frac{d\hat{L}_M}{d\hat{L}} - 1 = \frac{\tilde{L}_A a_\Lambda}{\kappa}\left[(1-\theta)\left(\frac{1}{\frac{1}{1-\varepsilon}} - \frac{1}{\frac{1}{1-\rho}(1-\chi)}\right) - \frac{\theta}{\frac{1}{1-\rho}}\left(1 - \frac{1}{\frac{1}{1-\varepsilon}}\right)\right]$$

a_j – farm income shares, $\chi = \bar{a}\tilde{L}/\tilde{A}$, $\theta = p\tilde{M}/\tilde{Y}_M$

- Farm Productivity

$$\frac{d\hat{L}_M}{d\hat{B}_A} = \frac{\tilde{L}_A(1-\theta)}{\kappa}\left[\frac{1}{\frac{1}{1-\rho}(1-\chi)} - 1\right]$$

Employment Share in Manufacturing: Analytical Results

- Population Size

$$\frac{d\hat{L}_M}{d\hat{L}} - 1 = \frac{\tilde{L}_A a_\Lambda}{\kappa}\left[(1-\theta)\left(\frac{1}{\frac{1}{1-\varepsilon}} - \frac{1}{\frac{1}{1-\rho}(1-\chi)}\right) - \frac{\theta}{\frac{1}{1-\rho}}\left(1 - \frac{1}{\frac{1}{1-\varepsilon}}\right)\right]$$

a_j – farm income shares, $\chi = \bar{a}\tilde{L}/\tilde{A}$, $\theta = p\tilde{M}/\tilde{Y}_M$

- Farm Productivity

$$\frac{d\hat{L}_M}{d\hat{B}_A} = \frac{\tilde{L}_A(1-\theta)}{\kappa}\left[\frac{1}{\frac{1}{1-\rho}(1-\chi)} - 1\right]$$

- Manufacturing Productivity

$$\frac{d\hat{L}_M}{d\hat{B}_M} = \frac{\tilde{L}_A}{\kappa s_L}\left[\left(1-\theta+\frac{\theta}{\frac{1}{1-\rho}}\right)(1-a_K) + \frac{a_K}{1-\varepsilon}\frac{(1-\theta\chi)}{\frac{1}{1-\rho}(1-\chi)} - \frac{1}{\frac{1}{1-\rho}}\right]$$

Quantitative Investigation

- Calibrate the model in steady state to the (stable) 1600-1650 period.

Quantitative Investigation

- Calibrate the model in steady state to the (stable) 1600-1650 period.
- Estimate Factor Specific Technological Progress in both sectors.

Quantitative Investigation

- Calibrate the model in steady state to the (stable) 1600-1650 period.
- Estimate Factor Specific Technological Progress in both sectors.
- Feed in all the exogenous changes simultaneously into the model (population, technology, trade). Compare the model dynamics to the data.

Quantitative Investigation

- Calibrate the model in steady state to the (stable) 1600-1650 period.
- Estimate Factor Specific Technological Progress in both sectors.
- Feed in all the exogenous changes simultaneously into the model (population, technology, trade). Compare the model dynamics to the data.
- To assess the importance of population change – shut it down in the benchmark model.

Calibration Strategy

- Calibrate to match empirical moments during 1600-1650.

Calibration Strategy

- Calibrate to match empirical moments during 1600-1650.
- Keep parameters ρ and ε as well as the "target" $\chi = \bar{a}\tilde{L}/\tilde{A}$ free.

Calibration Strategy

- Calibrate to match empirical moments during 1600-1650.
- Keep parameters ρ and ε as well as the "target" $\chi = \bar{a}\tilde{L}/\tilde{A}$ free.
- Perform extensive sensitivity analysis with respect to ρ, ε, χ.

Calibration

Others to ensure a match with the following targets, calculated from the 1550-1650 data:

$$[i] : r + 1 - \delta = 1.04^5, \quad [ii] : \frac{p_A Y_A}{Y} = 0.65$$
$$[iii] : \frac{L_A}{L} = 0.59, \quad [iv] : s_L = 0.75$$
$$[v] : a_L = .55, \quad [vi] : a_K = 0.15$$
$$[vii] : \delta = 1 - 0.95^5 \quad [viii] : M = 0$$
$$[ix] : \chi$$

Benchmark Calibrated Parameters

- $\gamma = -1$ to ensure the elasticity of intertemporal substitution is 2

Benchmark Calibrated Parameters

- $\gamma = -1$ to ensure the elasticity of intertemporal substitution is 2
- Normalizations: $B_{A,1650} = 1$, $\Lambda = 100$

Benchmark Calibrated Parameters

- $\gamma = -1$ to ensure the elasticity of intertemporal substitution is 2
- Normalizations: $B_{A,1650} = 1$, $\Lambda = 100$
- Free critical parameters ρ, ε and critical target χ are chosen for the benchmark model as

Benchmark Calibrated Parameters

- $\gamma = -1$ to ensure the elasticity of intertemporal substitution is 2
- Normalizations: $B_{A,1650} = 1$, $\Lambda = 100$
- Free critical parameters ρ, ε and critical target χ are chosen for the benchmark model as
  - $\varepsilon = -3$ (Allen 2008, and other economic historians)

Benchmark Calibrated Parameters

- $\gamma = -1$ to ensure the elasticity of intertemporal substitution is 2
- Normalizations: $B_{A,1650} = 1$, $\Lambda = 100$
- Free critical parameters ρ, ε and critical target χ are chosen for the benchmark model as
  - $\varepsilon = -3$ (Allen 2008, and other economic historians)
  - $\rho = -1$ (complementarity)

Benchmark Calibrated Parameters

- $\gamma = -1$ to ensure the elasticity of intertemporal substitution is 2
- Normalizations: $B_{A,1650} = 1$, $\Lambda = 100$
- Free critical parameters ρ, ε and critical target χ are chosen for the benchmark model as
  - $\varepsilon = -3$ (Allen 2008, and other economic historians)
  - $\rho = -1$ (complementarity)
  - $\chi = 0.3$

Benchmark Calibrated Parameters

Table: Calibrated Parameter Values

$B_{A,1650} = 1,\ b_{A,1650} = 0.2,\ \ c_{A,1650} = 0.5,\ \ \varepsilon = -3$
$B_{M,1600} = 1.7\ \ \upsilon = 0.25$
$\bar{a} = 0.185,\ \ \alpha = 0.53,\ \ \beta = 0.82,\ \ \rho = -1,\ \ \gamma = -1$
$\delta = 0.12,\ \ \Lambda = 100,\ \ L_{1650} = 156,\ \ M_{1650} = 0$

Technological Progress Estimation

Need to estimate $\{B_{M,t}\}$ and $\{B_{A,t}, b_t, c_t\}$.

- Manufacturing Production

$$B_{M,t} = \left(\frac{r_t}{\upsilon}\right)^{\upsilon} \left(\frac{w_t}{1-\upsilon}\right)^{1-\upsilon}$$

Technological Progress Estimation

Need to estimate $\{B_{M,t}\}$ and $\{B_{A,t}, b_t, c_t\}$.

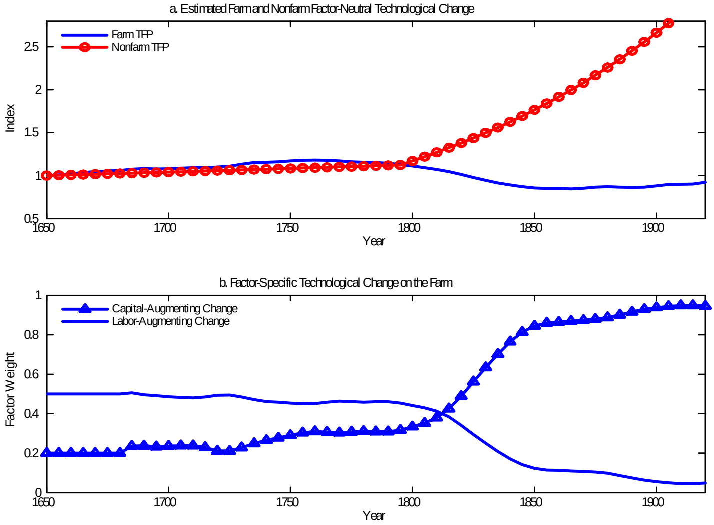
- Manufacturing Production

$$B_{M,t} = \left(\frac{r_t}{\upsilon}\right)^{\upsilon} \left(\frac{w_t}{1-\upsilon}\right)^{1-\upsilon}$$

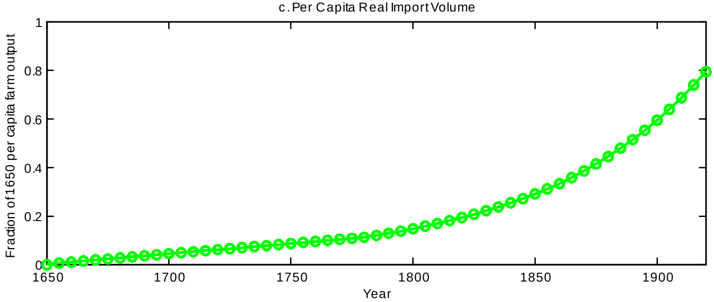
- Farm Production

$$\begin{aligned}
\frac{b_{A,t}}{c_{A,t}} &= \frac{r_t}{w_t} \left(\frac{K_{A,t}}{L_{A,t}}\right)^{1-\varepsilon}, \\
\frac{1 - b_{A,t} - c_{A,t}}{c_{A,t}} &= \left(\frac{\Lambda}{L_{A,t}}\right)^{1-\varepsilon} \frac{\rho_{\Lambda t}}{w_t}, \\
B_{A,t} &= \frac{w_t}{p_t} \left[\frac{s_{L,t}^{1-\varepsilon}}{c_{A,t}}\right]^{\frac{1}{\varepsilon}}.
\end{aligned}$$

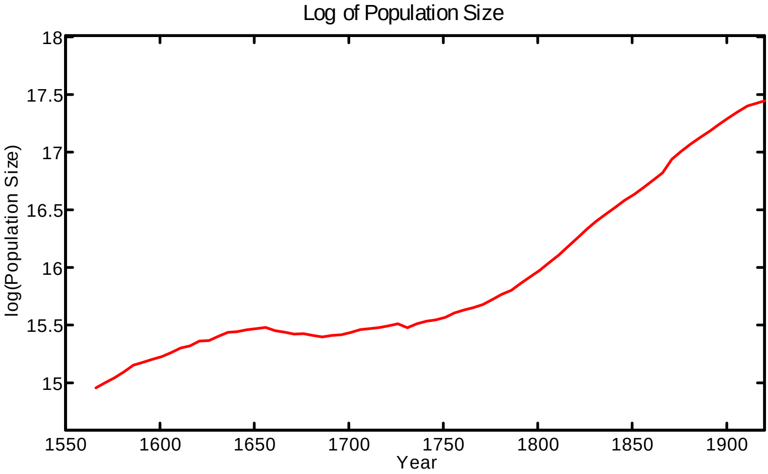
Exogenous Changes: Technology and Trade



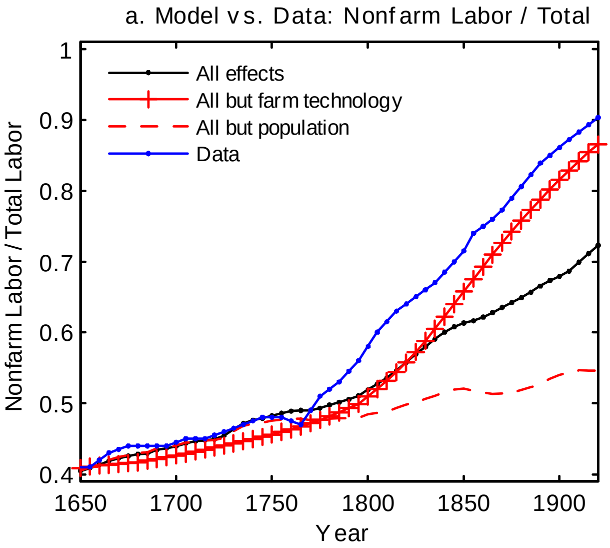
Exogenous Changes: Technology



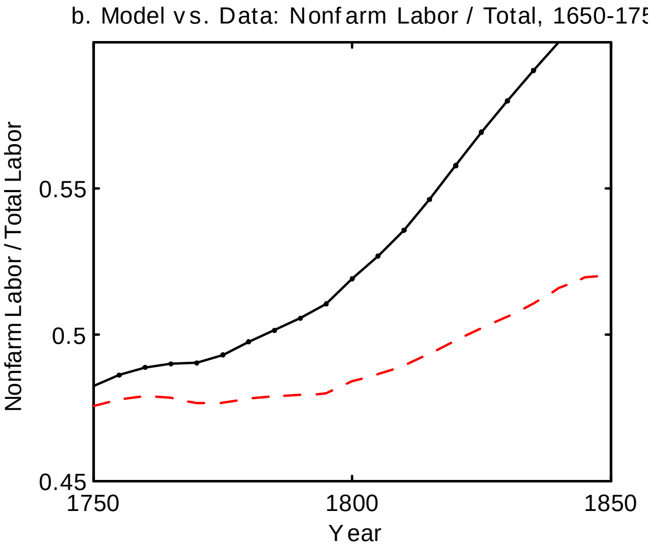
Exogenous Changes: Population Size



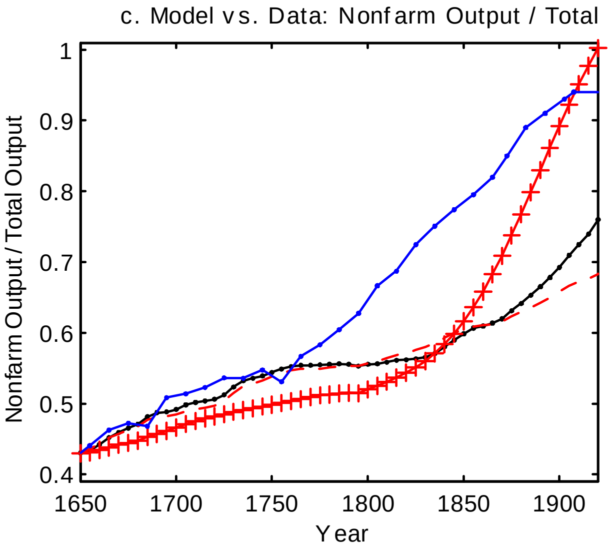
Model vs. Data



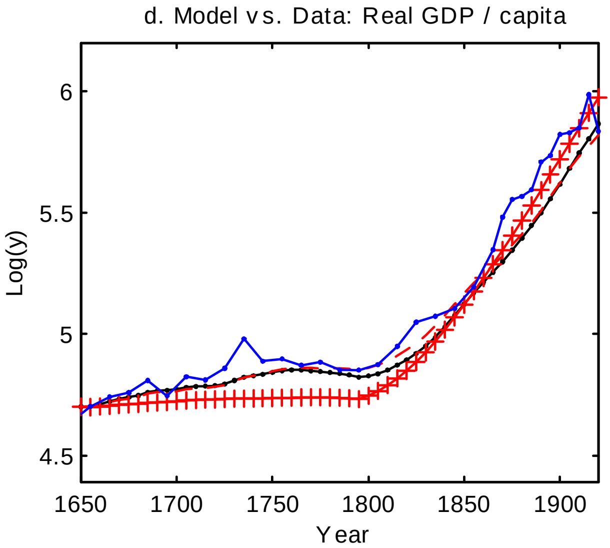
Model vs. Data



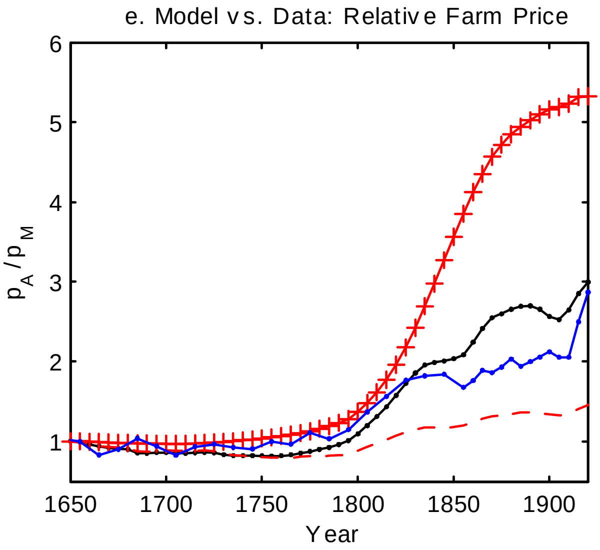
Model vs. Data



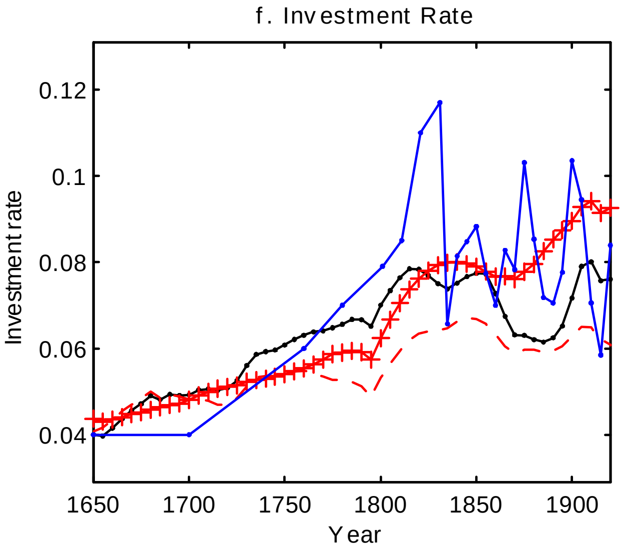
Model vs. Data



Model vs. Data



Model vs. Data



Main Results, Benchmark Model

$(\rho = -1)$

	L_m/L	Y_m/Y	log y	I/Y	p
% ch. Data, 1650-1920	**120**	**124**	**26**	**110**	**195**
% acct. for by the main experiment	**65**	**62**	**96**	**82**	**102**
farm technology effect	-42	-74	-9	-24	-117
population effect	**56**	**23**	**4**	**45**	**77**
% of 1650-1750 ch. acct. for by					
the main experiment	112	87	95	139	270
farm technology effect	40	40	74	55	121
population effect	10	5	-3	37	-8
% of 1750-1920 ch. acct. for by					
main experiment	57	55	97	47	123
farm technology effect	-80	-155	-24	-186	-55
population effect	**70**	**32**	**5**	**51**	**69**

Main Results, Increase Substitutability

$(\rho = -0.1)$

	L_m/L	Y_m/Y	$\log y$	I/Y	p
% ch. Data, 1650-1920	**120**	**124**	**26**	**110**	**195**
% acct. for by the main experiment	**70**	**75**	**97**	**86**	**80**
farm technology effect	-36	-55	-10	-28	-128
population effect	**50**	**27**	**4**	**44**	**72**
% of 1650-1750 ch. acct. for by					
the main experiment	95	73	96	140	247
farm technology effect	25	23	74	52	121
population effect	14	9	-3	37	-8
% of 1750-1920 ch. acct. for by					
the main experiment	66	81	97	52	96
farm technology effect	-56	-88	-24	-179	-64
population effect	**58**	**32**	**5**	**46**	**63**

Sensitivity: % of Empirical Change Accounted for

				1650-1920				1750-1920			
ε	$\frac{\bar{a}}{a}$	ρ	*model*	$\frac{L_m}{L}$	$\frac{Y_m}{Y}$	$\frac{I}{Y}$	p	$\frac{L_m}{L}$	$\frac{Y_m}{Y}$	$\frac{I}{Y}$	p
-3	0.3	-2	total	63	56	77	112	52	45	41	136
			pop.	58	21	43	79	76	32	48	72
-3	0.3	-1	total	65	62	82	102	56	55	47	123
			pop.	56	23	45	77	70	32	51	69
-3	0.5	-1	total	68	67	97	92	55	56	53	116
			pop.	44	14	46	76	57	19	56	67
-3	0.3	0.5	total	87	106	153	54	96	144	142	64
			pop.	42	31	65	60	41	30	74	51
-1	0.3	-1	total	52	57	94	69	38	50	60	83
			pop.	41	19	52	68	57	25	57	59

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:
  - Low substitutability between factors of production on the farm, with land given in fixed supply

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:
    - Low substitutability between factors of production on the farm, with land given in fixed supply
    - Absorption of labor in the nonfarm sector – determined by

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:
  - Low substitutability between factors of production on the farm, with land given in fixed supply
  - Absorption of labor in the nonfarm sector – determined by
    1. demand conditions substitutability of farm and nonfarm products

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:
    - Low substitutability between factors of production on the farm, with land given in fixed supply
    - Absorption of labor in the nonfarm sector – determined by
        1. demand conditions substitutability of farm and nonfarm products
        2. was greatly facilitated by the expansion of trade

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:
  - Low substitutability between factors of production on the farm, with land given in fixed supply
  - Absorption of labor in the nonfarm sector – determined by
    1. demand conditions substitutability of farm and nonfarm products
    2. was greatly facilitated by the expansion of trade
- Farm technology is important for the earlier period, 1650-1750

Conclusions

- Population size effect appears to be quantitatively important in accounting for the structural development of England, especially since 1750.
- The main mechanism:
  - Low substitutability between factors of production on the farm, with land given in fixed supply
  - Absorption of labor in the nonfarm sector – determined by
    1. demand conditions substitutability of farm and nonfarm products
    2. was greatly facilitated by the expansion of trade
- Farm technology is important for the earlier period, 1650-1750
- Nonfarm technology drives output dynamics